

Squeezing MOND into a Cosmological Scenario

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Abstract

Explaining the effects of dark matter using modified gravitational dynamics (MOND) has for decades been both an intriguing and controversial possibility. By insisting that the gravitational interaction that accounts for the Newtonian force also drives cosmic expansion, one may kinematically identify which cosmologies are compatible with MOND, without explicit reference to the underlying theory so long as the theory obeys Birkhoff's law. Using this technique, we are able to self-consistently compute a number of quantities of cosmological interest. We find that the critical acceleration a_0 must have a slight source-mass dependence ($a_0 \sim M^{1/3}$) and that MOND cosmologies are naturally compatible with observed late-time expansion history and the contemporary cosmic acceleration. However, cosmologies that can produce enough density perturbations to account for structure formation are contrived and fine-tuned. Even then, they may be marginally ruled out by evidence of early ($z \sim 20$) reionization.

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I. INTRODUCTION

That approximately ninety percent of the matter in the universe is composed of some as of yet unspecified material is an unsettling prospect, especially as an increasingly coherent picture of cosmology emerges. And while there are several well-motivated candidates for dark matter, its sole function is to provide gravitational ballast by offering supplemental mass in astrophysical and cosmological settings where the accounting of visible matter falls short of gravitational expectations or requirements. Replacing dark matter with a modification of the laws of gravity (as encoded in the paradigm of Modified Newtonian Dynamics, MOND) has been for decades both an intriguing and a controversial alternative [1,2].

Nevertheless, the dark matter paradigm works quite well and offers a litany of successes in its contribution to the standard cosmological model. In contrast, MOND, though not totally silent, is largely inarticulate concerning cosmology: it is a paradigm designed to address galaxy rotation curves, there exists no satisfactory underlying theory, and there is some difficulty in incorporating it into a believable cosmological scenario [3–6].

In this paper, we reexamine the possibility of folding MOND into a cosmological model under the premise that the same gravitational interactions that manifest themselves in a modified Newtonian force are also responsible for cosmological evolution. In a previous paper [7] with Scoccimarro, we devised a technique by which one may kinematically derive a unique Schwarzschild-like metric for a modified gravity theory from a specified, nonstandard homogeneous cosmology. Again, the presumption exploited was that cosmology is driven exclusively by gravitational self-interactions of the constituent matter, rather than by some unknown energy-momentum component such as dark energy. This correspondence between the metric and cosmology can be made completely without reference to the fundamental modified-gravity theory, using only the assumption that the underlying theory respects Birkhoff’s law. The procedure is simply the generalization of the classic description of how one recovers the Friedmann equation from the Newtonian force law, but generalized to a full metric theory and beyond Einstein gravity.

We apply the same technique here to ascertain which cosmologies are compatible with MOND, allowing us to identify a full Schwarzschild-like metric and providing a self-consistent framework to perform calculations of interest in MOND cosmology. We begin by briefly reviewing the prescription for the full metric consistent with homogeneous cosmologies and apply that prescription to determine both the Schwarzschild-like metric and the modified Friedmann equation of MOND. Then we examine the class of cosmologies consistent with the MOND force law and reveal that there are potentially insurmountable difficulties that arise when one wishes to incorporate MOND into a consistent cosmological scenario.

II. METRIC MOND AND LOCAL GRAVITY

Let us quickly review the technique developed in Ref. [7] where one infers the Schwarzschild metric from an arbitrary cosmology, to see how one might apply it in reverse and devise a cosmology consistent with Modified Newtonian Dynamics. Consider a homogeneous cosmology described by the line element

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j , \quad (2.1)$$

with some specified scale factor evolution, $a(t)$. If this universe is matter-dominated (such that the Universe is filled homogeneously by matter whose density obeys the relationship $\rho(t) \sim a^{-3}$), one is faced with either of two possibilities. First, one may believe that Einstein gravity is correct and that the cosmology is driven by some unseen additional energy-momentum components – dark matter and dark energy. The other possibility is that the matter we see is the only energy-momentum component and that one needs to alter gravitational dynamics in a specific way to achieve the observed cosmic expansion history, $a(t)$. We follow the latter possibility.

It is convenient to represent the given scale factor evolution as the solution to some alternative Friedmann equation:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = H_0^2 g(x) , \quad (2.2)$$

where $x = \frac{8}{3}\pi G\rho/H_0^2$ is a dimensionless parameter, G is Newton's constant, and H_0 is today's Hubble scale. The function $g(x)$ is determined by the given $a(t)$. If one requires that the fundamental gravitational theory respects Birkhoff's law, then one can uniquely determine the metric of a spherically symmetric source [7]. That metric is described by the line element

$$ds^2 = g_{00}(r)dt^2 - g_{rr}(r)dr^2 - r^2 d\Omega , \quad (2.3)$$

with

$$g_{00}(r) = g_{rr}^{-1} = 1 - r^2 H_0^2 g(r_g/r^3 H_0^2) . \quad (2.4)$$

Here $r_g = 2GM$, is the usual Schwarzschild radius of a matter source of mass M . Note that the form of the metric components is completely determined by $a(t)$, and in particular, that r_g and r can only appear in the metric in a specific combination. This point will be important when we consider how to apply this connection between cosmology and the Schwarzschild-like metric to MOND.

In MOND [1] the gravitational acceleration exerted by a body of mass M obeys the relationship:

$$a = \begin{cases} -\frac{1}{2} \frac{dg_{00}}{dr} = -GM r^{-2} & |a| > a_0 \\ \sim -r^{-1} & |a| < a_0 \end{cases} \quad (2.5)$$

for some critical acceleration, a_0 . If we insist on a form for modified gravity which is compatible with a homogeneous cosmology and Birkhoff's law, its Schwarzschild-like metric must be of the form Eq. (2.4). The form for $g(x)$ compatible with Eq. (2.5) is

$$g(x) = \begin{cases} x + c_1 x^{2/3} & \text{Einstein } (x > x_c) \\ \beta x^{2/3} \ln x + c_2 x^{2/3} & \text{MOND } (x < x_c) \end{cases}, \quad (2.6)$$

for some constant parameters, β , c_1 , and c_2 , yielding

$$a = \begin{cases} -\frac{1}{2} \frac{r_g}{r^2} & |a| > a_0 \\ -\frac{3\beta}{2} \frac{(r_g H_0)^{2/3}}{r} & |a| < a_0 \end{cases}, \quad (2.7)$$

where the critical MOND acceleration, a_0 , is

$$a_0 = H_0 \left[9\beta^2 (r_g H_0)^{1/3} \right]. \quad (2.8)$$

Observationally, we choose $\beta \approx 15$ so that for source masses the size of large galaxies ($M \sim 10^{11} M_\odot$), the critical acceleration is $a_0 \approx \frac{1}{6} H_0$, corresponding to $x_c \approx 7 \times 10^4$. A relationship between c_1 and c_2 exists to ensure that $g(x)$ is continuous across the transition at $a = a_0$:

$$c_1 = c_2 + 3\beta [\ln(3\beta) - 1], \quad (2.9)$$

and the remaining constant represent an arbitrary choice in zero-point energy for the Newtonian potential.¹

The form Eq. (2.7) for the MOND gravitational acceleration is slightly different than the form typically considered, i.e., one where $a_0 = \frac{1}{6} H_0$ is a universal constant. Compatibility with a homogeneous cosmology compels us to choose a form where the critical acceleration has a weak dependence on the source mass, $a_0 \sim M^{1/3}$. This mass dependence is not a serious amendment to the MOND paradigm, and indeed a well-motivated mass dependence

¹Although the c_1 and c_2 -terms do not affect the Newtonian acceleration, they do have a nontrivial affect on cosmology, simulating curvature-type terms in the Friedmann equation even though the cosmology is explicitly spatially-flat (see Eq. (2.1)). Moreover, these terms have a effect on g_{rr} (see Eq. (2.4)), manifesting in nontrivial, though immeasurably small, effects on gravitational lensing and the post-Newtonian parameter, γ .

may actually benefit the modified-gravity scenario. Galaxy clusters appear to stray from the original MOND parametrization [2], such that uncomfortably large light-to-mass ratios are necessary to bring objects of mass scales $\mathcal{O}(10^{14}M_\odot)$ into accord with MOND. In our prescription, Eq. (2.7), objects more massive than galaxies would exhibit effective dark matter halos somewhat heavier than those predicted by traditional MOND, consistent with what seems to be required by observations.

Let us summarize. By requiring that the underlying fundamental gravity theory that provides MOND Newtonian accelerations are compatible with both homogeneous cosmologies and Birkhoff’s law, we may construct the function $g(x)$ found in Eq. (2.6) which determines both the modified Friedmann equation, Eq. (2.2), and the full Schwarzschild-like metric of a spherical mass source, Eq. (2.4), avoiding explicit reference to the details of the fundamental theory. With these two governing relationships, we may now articulate a whole host of important properties of cosmological interest. For example, we may compute modifications of planetary ephemeris, gravitational lensing, growth of density perturbations (both linear and nonlinear), and the late-time integrated Sachs–Wolfe (ISW) effect on the cosmic microwave background. Details of these calculations for arbitrary $g(x)$ are given in Ref. [7] Here we focus particularly on accommodating late-time acceleration into MOND, and on the growth of fluctuations.

III. MOND COSMOLOGY

Beginning with a function $g(x)$ given by Eq. (2.6) and consistent with MOND Newtonian accelerations, Eq. (2.7), one can articulate a MOND cosmology. The quantity x that appears in the modified Friedmann equation, Eq. (2.2), may be interpreted as $\Omega_b(t)$ assuming that the matter content in the universe is dominated by baryons. (At earlier times one would require some MOND description of the self-gravity of radiation.) One can then immediately associate x with a redshift using the relationship $x = (1 + z)^3 \Omega_b^{\text{today}}$. Let us investigate the cosmology in stages, beginning with the Einstein, large- x , stage.

A. Early cosmology: The CMB

The transition from from Einstein to MOND takes place in galaxies at $x \sim 7 \times 10^4$, or correspondingly, taking $\Omega_b^{\text{today}} \approx 0.04$ [8], cosmologically at a redshift $z \sim 120$. Thus, cosmology at redshifts $z \gtrsim 120$ follows the ordinary GR Friedmann equation. But if the matter content of the universe is solely baryonic, then matter–radiation equality occurs at $z \sim 600$ whereas recombination still occurs at $z \sim 1100$, implying that recombination is *before* radiation-matter equality rather than after it as in conventional dark-matter cosmology.

This observation corroborates the approach taken in prior work regarding how MOND affects the cosmic microwave background (CMB) [12,13]. The acoustic oscillations that appear in the CMB anisotropy must, as expected, be driven in an almost purely baryonic scenario. This prior work claims that MOND not only survives this drastic discrepancy from the standard cosmological model, but that some ratios of CMB peak heights indeed favor MOND. Since our prescription applies strictly only during the matter-dominated epoch, it has nothing to contribute to the understanding of MOND physics at the epoch of last scattering, although it can be used to make predictions about the late-time integrated Sachs–Wolfe (ISW) effect.

B. Late cosmology: Recollapse, expansion and acceleration

Looking at the form of Eq. (2.6), it is clear one cannot extend MOND force-law to arbitrarily small x , or density. Eventually, the Hubble parameter, $H = \dot{a}/a$ vanishes and the universe recollapses, regardless of the choice one makes for c_2 . The result is intuitive if one imagines cosmology as evolving classically on the Newtonian potential at some fixed energy dictated by c_2 . The scale factor, a , is proportional to the position of a test particle on that potential. The MOND part of the potential is logarithmic, implying that every comoving trajectory eventually has a turning point for any choice of initial energy (i.e., choice of c_2).

Thus, there must be a sufficiently small x where MOND behavior ceases to predominate. We are guided here by the data which teaches us that:

1. Our Hubble expansion rate is currently $H \equiv H_0 \simeq 70$ km/s. (A value 20 or 30% smaller than this would not change these arguments materially.)
2. We are currently undergoing acceleration in our cosmic expansion [9,10] with $\ddot{a}/a \sim H_0^2$.
3. The expansion before $z \sim 1.7$ was decelerating [11].

We can accommodate these considerations by modifying $g(x)$ of Eq. (2.6) in the following way:

$$g(x) = \begin{cases} x + 3\beta x^{2/3} [\ln(3\beta) - 1] & x \gtrsim (3\beta)^3 & \text{(Einstein),} \\ \beta x^{2/3} \ln(1+x) & 0.1 \lesssim x \lesssim (3\beta)^3 & \text{(MOND),} \\ \Omega_\Lambda & x \lesssim 0.1 & \text{("Dark Energy"),} \end{cases} \quad (3.1)$$

where $g(x) = \Omega_\Lambda \approx 0.7$ is equivalent to a cosmological constant. Figure 1 shows these different regimes and a possible smooth interpolation. It is interesting that such a simple modification may be accommodated. If β were an order-of-magnitude larger or smaller, one could not extend the MOND regime all the way to the deceleration-acceleration transition and still be able to maintain both $H \sim H_0$ as well as $\ddot{a}/a \sim H_0^2$.

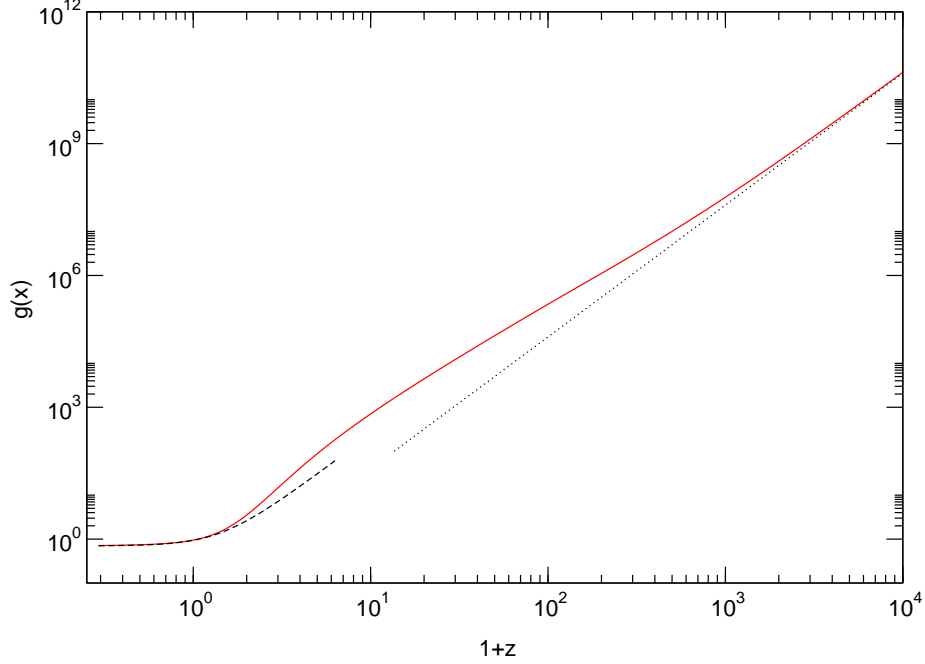


FIG. 1. The function $g(x)$ versus $1+z$ where $x = (1+z)^3 \Omega_b^{\text{today}}$. The dotted line is $g(x) = x$, the asymptotic Einstein behavior at high redshifts. The dashed line is $g(x) = \Omega_\Lambda + 6x$, which reproduces the late-time expansion history of a standard dark-matter model. The interpolating $x^{2/3} \ln(1+x)$ dependence is the MOND regime.

C. Late cosmology: Linear perturbation growth

Equations (2.2) and (3.1) represent the full modified Friedmann equation from matter-radiation equality to the present time, including the onset of today's cosmic acceleration. We may now proceed to compute the growth of linear perturbations in this cosmology. Such a computation is important because we require sufficient density perturbation growth to seed the observed structure in the Universe. The evolution of linear density perturbations for the class of theories under consideration take a simple closed form. Take a uniform overdensity in a localized spherical region such that

$$\rho(t) = \bar{\rho}(t) [1 + \delta(t)] , \quad (3.2)$$

where $\bar{\rho}$ is the background matter density that follows cosmological evolution. Parameterizing time-evolution using $x = 8\pi G \bar{\rho} / 3H_0^2$, the growing perturbation mode $\delta(x)$ goes as [7] (see also [14]):

$$\delta(x) = \frac{5A}{6} g^{1/2}(x) \int_x^\infty \frac{dy}{y^{1/3} g^{3/2}(y)} , \quad (3.3)$$

where A is an overall normalization. Because matter is predominantly baryons, the normalization is fixed by requiring that perturbations are restricted to be $\delta \sim \text{few} \times 10^{-5}$ at recombination. Figure 2 depicts the evolution of $\delta(x)$ for a smooth interpolation of Eq. (3.1).

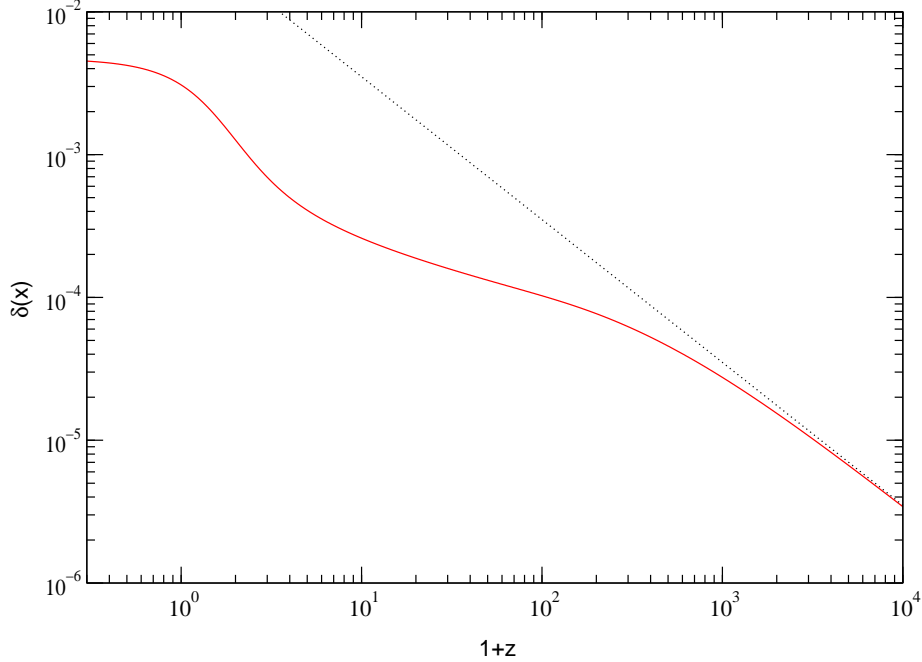


FIG. 2. Baryon density perturbation, $\delta(x)$ versus $1+z$ for the function $g(x)$ depicted in Fig. 1, normalized to be consistent with CMB anisotropy amplitude at recombination. The dotted line represents the growth in $\delta(x)$ if it were to follow Einstein gravity.

In pure matter-domination for Einstein FRW, $\delta = Ax^{-1/3}$, or in other words, δ grows like the scale factor $a(t)$. Even if growth were as large as this, given the normalization required at recombination, the growth of density perturbations would be insufficient to account for the observed structure formation. Moreover, in each of the three regimes in Eq. (3.1), growth is slower than that given benchmark, $\delta < a(t)$. In the early Einstein phase and during the MOND phase, scale factor evolution looks as if it were curvature-dominated. Growth must take place before the end of the MOND phase.

It may seem counterintuitive that growth is suppressed during the MOND regime, given that in such a regime the self-gravitation of overdensities should be enhanced. But one must recall that the same stronger gravity also drives a faster cosmology, which in turn suppresses perturbation growth. Ultimately, this latter effect wins out. This poses a significant difficulty for cosmological incarnations of MOND.

D. Late cosmology: Tuning the late-time potential

There is a way to avoid this difficulty, but a specially-tailored force law is required. From galaxy rotation curves, we require that MOND need only be valid up to radii $r \sim 70$ kpc for galaxy masses $M \sim 10^{11} M_\odot$ [15,16]. This distance and mass scale corresponds to $x \sim 600$ or, in MOND cosmology, to a redshift $z \sim 25$. If a recent observation of a Gunn–Peterson trough [17] is a signal that galaxies formed near $z \sim 6$ (or $x \sim 14$), then there is a narrow

window in x , between 14 and 600, where little is known observationally and where one can carefully manipulate $g(x)$ to achieve sufficient growth in density perturbations to create galaxies, yet still maintain the MOND paradigm.

To achieve the required growth, somewhere in this range of x , the function $g(x)$ must dip very close to zero and then rises again above $\mathcal{O}(1)$ to accommodate SNIA constraints on contemporary expansion history. From Eq. (3.3) one sees that near a minimum where $g(x) = g(x_0) + \frac{1}{2}g''(x_0)(x - x_0)^2$

$$\delta \sim \frac{A}{x_0^{1/3}} \frac{1}{\sqrt{g(x_0)g''(x_0)}}. \quad (3.4)$$

When $g(x_0)$ is close to zero, arbitrarily large growth in $\delta(x)$ can occur. The cosmology in this regime loiters, the expansion almost stops and near this critical unstable point in the potential, small variations in density amplify. For a $M \sim 10^{11}M_\odot$ source mass, this fine-tuned dip in $g(x)$ corresponds to Newtonian gravity becoming *repulsive* in a region $r \sim 70 \rightarrow 300$ kpc to generate the large perturbations and then becoming attractive again before $r \sim 600$ kpc to account for today's cosmology.

But even this possible resolution is a tenuous one. WMAP observations of the CMB suggests that reionization starts as early as $z \approx 20_{-9}^{+10}$ [8], and that growth in perturbations must occur before that redshift. The window for a possible excursion in $g(x)$ then becomes exceedingly small, casting doubt that a MOND cosmology can viably create the universe we see today.

IV. CONCLUDING REMARKS

In this paper, we provided a self-consistent framework where we could assess which cosmologies were compatible with Modified Newtonian Dynamics (MOND), exploiting techniques developed in prior work [7]. Our starting point was the MOND paradigm that the contents of the universe are what we see, and that these alone drive the dynamics of the cosmic expansion. We find that in order for MOND to exhibit homogeneous cosmologies, the critical MOND acceleration a_0 , which separates Einstein behavior from a modified Newtonian force, must have a slight source-mass dependence ($a_0 \sim M^{1/3}$).

With that mild amendment, we found that MOND cosmologies are naturally compatible with observed late-time expansion histories and late-time cosmic acceleration. However, those natural cosmologies cannot produce enough growth of density perturbations to account for structure formation. Two effects contribute: first, because matter is almost exclusively baryonic, matter perturbations are constrained to be $\mathcal{O}(10^{-5})$ at recombination; furthermore, during those redshifts where MOND behavior dominates, growth of perturbations is

suppressed, rather than enhanced as one might have expected. Although gravity is stronger than usual in this regime, the potential enhanced self-gravitation of density fluctuations is beaten by the faster expansion at a given redshift required if the Hubble parameter is to have its observed value today despite the stronger gravity that MOND predicts.

One may circumvent this difficulty by envisioning a loitering phase, arising from a drastic weakening of gravity at a selected value of x , and chosen to correspond to part of redshift history where little is known ($z \sim 6 \rightarrow 25$) – as the the cosmic expansion stalls, the self-gravitation of perturbations proceeds unhindered. These cosmologies are contrived and fine-tuned, and may even be marginally ruled out by evidence of early ($z \sim 20$) reionization. Such machinations cast doubt on the possibility that MOND cosmology can viably lead to the universe we observe today.

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REFERENCES

- [1] M. Milgrom, *Astrophys. J.* **270**, 365 (1983).
- [2] R. H. Sanders and S. S. McGaugh, *Annual Review of Astronomy and Astrophysics* Sep 2002, Vol. 40: 263-317 [arXiv:astro-ph/0204521].
- [3] J. E. Felten, *Astrophys. J.* **286**, 3 (1984).
- [4] S. McGaugh, arXiv:astro-ph/9812327.
- [5] R. H. Sanders, *Astrophys. J.* **560**, 1 (2001).
- [6] A. Nusser, arXiv:astro-ph/0109016.
- [7] A. Lue, R. Scoccimarro and G. Starkman, arXiv:astro-ph/0307034.
- [8] C. L. Bennett *et al.*, *Astrophys. J. Suppl.* **148**, 1 (2003).
- [9] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *Astrophys. J.* **517**, 565 (1999).
- [10] A. G. Riess *et al.* [Supernova Search Team Collaboration], *Astron. J.* **116**, 1009 (1998).
- [11] N. Benitez *et al.* [Supernova Search Team Collaboration], *Astrophys. J.* 577 (2002) L1-L4
- [12] S. S. McGaugh, *Astrophys. J.* **523**, L99 (1999).
- [13] S. S. McGaugh, *Astrophys. J.* **541**, L33 (2000).
- [14] T. Multamaki, E. Gaztañaga and M. Manera, arXiv:astro-ph/0303526.
- [15] R. H. Sanders, *Astrophys. J.* **473**, 117 (1996)
- [16] W. J. G. de Blok and S. S. McGaugh, *Astrophys. J.* **508**, 132 (1998).
- [17] R. H. Becker *et al.* [SDSS Collaboration], *Astron. J.* **122**, 2850 (2001).